1. Find all strings in \( L((a + b)^*b(a + ab)^*) \) of length less than four.

2. Does the expression \(((0 + 1)(0 + 1)^*)^*00(0 + 1)^*\) denote the language in Example 3.5?

3. Show that \( r = (1 + 01)^*(0 + 1)^* \) also denotes the language in Example 3.6. Find two other equivalent expressions.

4. Find a regular expression for the set \( \{a^n b^m : n \geq 3, m \text{ is even} \} \).

5. Find a regular expression for the set \( \{a^n b^m : (n + m) \text{ is even} \} \).

6. Give regular expressions for the following languages.
   (a) \( L_1 = \{a^n b^m, n \geq 4, m \leq 3 \} \).
   (b) \( L_2 = \{a^n b^m : n < 4, m \leq 3 \} \).
   (c) The complement of \( L_1 \).
   (d) The complement of \( L_2 \).

7. What languages do the expressions \((\emptyset)^*\) and \(a\emptyset\) denote?

8. Give a simple verbal description of the language \( L((aa)^*b(aa)^* + a(aa)^*ba(aa)^*) \).

9. Give a regular expression for \( L^R \), where \( L \) is the language in Exercise 1.

10. Give a regular expression for \( L = \{a^n b^m : n \geq 1, m \geq 1, nm \geq 3 \} \).

11. Find a regular expression for \( L = \{ab^nw : n \geq 3, w \in \{a, b\}^+ \} \).

12. Find a regular expression for the complement of the language in Example 3.4.

13. Find a regular expression for \( L = \{vwu : v, w \in \{a, b\}^*, |v| = 2 \} \).

14. Find a regular expression for \( L = \{vwu : v, w \in \{a, b\}^*, |v| \leq 2 \} \).

15. Find a regular expression for \( L = \{w \in \{0, 1\}^* : w \text{ has exactly one pair of consecutive zeros} \} \).

16. Give regular expressions for the following languages on \( \Sigma = \{a, b, c\} \).
   (a) all strings containing exactly one \( a \),
   (b) all strings containing no more than three \( a \)’s,
   (c) all strings that contain at least one occurrence of each symbol in \( \Sigma \),
   (d) all strings that contain no run of \( a \)’s of length greater than two,
   (e) all strings in which all runs of \( a \)’s have lengths that are multiples of three.

17. Write regular expressions for the following languages on \( \{0, 1\} \).
   (a) all strings ending in 01,
   (b) all strings not ending in 01,
   (c) all strings containing an even number of 0’s,
   (d) all strings having at least two occurrences of the substring 00.
      (Note that with the usual interpretation of a substring, 000 contains two such occurrences),
   (e) all strings with at most two occurrences of the substring 00,
   (f) all strings not containing the substring 101.

18. Find regular expressions for the following languages on \( \{a, b\} \).
   (a) \( L = \{w : |w| \mod 3 = 0 \} \).
   (b) \( L = \{w : n_a(w) \mod 3 = 0 \} \).
   (c) \( L = \{w : n_a(w) \mod 5 > 0 \} \).

19. Repeat parts (a), (b), and (c) of Exercise 18, with $\Sigma = \{a, b, c\}$.

20. Determine whether or not the following claims are true for all regular expressions $r_1$ and $r_2$. The symbol $\equiv$ stands for equivalence of regular expressions in the sense that both expressions denote the same language.

   (a) $(r_1^*) \equiv r_1^*$.
   (b) $r_1^* (r_1 + r_2)^* \equiv (r_1 + r_2)^*$.
   (c) $(r_1 + r_2)^* \equiv (r_1^* r_2^*)^*$.
   (d) $(r_1 r_2)^* \equiv r_1^* r_2^*$.

21. Give a general method by which any regular expression $r$ can be changed into $\widehat{\alpha}$ such that $(L(r))^R = L(\widehat{\alpha})$.

22. Prove rigorously that the expressions in Example 3.6 do indeed denote the specified language.

23. For the case of a regular expression $r$ that does not involve $\lambda$ or $\varnothing$, give a set of necessary and sufficient conditions that $r$ must satisfy if $L(r)$ is to be infinite.

24. Formal languages can be used to describe a variety of two-dimensional figures. Chain-code languages are defined on the alphabet $\Sigma = \{u, d, r, l\}$, where these symbols stand for unit-length straight lines in the directions up, down, right, and left, respectively. An example of this notation is $urd$, which stands for the square with sides of unit length. Draw pictures of the figures denoted by the expressions $(rd)^*$, $(urdlru)^*$, and $(rulg)^*$.

25. In Exercise 24, what are sufficient conditions on the expression so that the picture is a closed contour in the sense that the beginning and ending points are the same? Are these conditions also necessary?

26. Find an nfa that accepts the language $L(aa^* (a + b))$.

27. Find a regular expression that denotes all bit strings whose value, when interpreted as a binary integer, is greater than or equal to 40.

28. Find a regular expression for all bit strings, with leading bit 1, interpreted as a binary integer, with values not between 10 and 30.